Joe Holbrook Memorial Math Competition

7th Grade

October 12, 2025

1. The first two terms are 0. So, the sum is $2 \times 5 + 5 = 10 + 5 = \boxed{15}$.
2. We just need to compute the remainder when 20 is divided by 3, which is 2. Since the remainder is less than the divisor, we can be sure that we cannot give more plushies. Thus, our answer is $\boxed{2}$.
3. Since David can write 5 questions every day, it will take him $\frac{40}{5} = \boxed{8}$ days.
4. Mark, Luke, and John are behind Matthew so there are $\boxed{3}$ people.
5. Emma's answer is $6 - 5 + 4 - 3 + 2 - 1 = 3$, while the correct answer is $6 - (5 + (4 - (3 + (2 - 1)))) = 6 - (5 + (4 - (3 + 1))) = 6 - (5 + (4 - 4)) = 6 - (5 + 0) = 6 - 5 = 1$. So the positive difference between the two is $3 - 1 = \boxed{2}$.
6. We see the first two digits 2 and 0 are fixed and the latter two are maximized in the year 2019 where the sum is $2 + 0 + 1 + 9 = \boxed{12}$.
7. He runs 5 miles 6 days a week and 10 miles one day a week for a total of $5 \times 6 + 10 = \boxed{40}$ miles a week.
8. For simplicity, let us convert everything to inches. We know the giant is 165 inches tall and that Arnav guessed it to be 149 inches. Since Arnav was off by 16 inches, Gabriele must also be off by 16 inches, but in the other direction. Thus, we add 16 to 165 and we get $\boxed{181}$.
9. Joe is six times faster than Alex while Bob is only twice as fast. Thus, Joe is three times faster than Bob, so it would take Bob $210 \cdot 3 = \boxed{630}$ minutes.
10. We have the prime factorizations $625 = 5^4$ and $2025 = 5^2 \times 3^4$, which means that the greatest common factor is $5^2 = \boxed{25}$.
11. Since 24 hours is equivalent to $24 \cdot 60$ minutes, our answer is $\frac{24 \cdot 60}{20} = \boxed{72}$.
12. The maximum distance between them occurs when Olivia and Esther are on opposite sides of BCA in a straight line. In that case, they are $9.0 + 6.5 = 15.5$ kilometers apart. The minimum distance occurs when they are on the same side of BCA in a straight line, making them $9.0 - 6.5 = 2.5$ kilometers apart. Therefore, the positive difference between the maximum and minimum possible distances is $15.5 - 2.5 = \boxed{13}$ kilometers.
13. At 7 o'clock, the hour hand points to 7, and it will do so every 12 hours. The remainder when 2025 is divided by 12 is 9, which means the hour hand will advance a total of 9 hours by the end. Starting at 7 and moving forward 9 hours on the clock, our answer is $\boxed{4}$.
14. The region that is swept is a circle, and the distance from the center to the vertex is the circle's radius. Thus, the area is $\pi r^2 = \pi(42)^2 = 1764\pi$, so our answer is $\boxed{1764}$.
15. Bob's painting is placed in a frame that forms a border that is 1 inch thick on all sides. Given that the painting itself has a length twice as long as its width and has a perimeter of 54 inches, find the area of the frame in square inches.
16. Note that $2^{22} \cdot 5^{10} = 2^{12} \cdot 2^{10} \cdot 5^{10} = 2^{12} \cdot 10^{10}$. Now we just have to find the number of digits that 2^{12} has, and add 10 extra digits for the 10 zeros that come with 10^10. Since $2^{12} = 4096$ has 4 digits, our original product has $4 + 10 = \boxed{14}$ digits.
17. We simply need to count how many of the 36 pairs of rolls he can achieve satisfy our condition. For example, we would first count $(1, 1), (1, 2), \ldots (1, 6)$, then count $(2, 1), (2, 2), (2, 4), (2, 6)$, and so on. This gives us a probability of $\frac{22}{36}$ for a final answer of $\boxed{22}$.

18. Using the clues, there is 1 option for the thousands digit, 10 options for the hundreds digit, 6 options for the tens digit, and 5 options for the ones digit. Thus, there are $1 \cdot 10 \cdot 6 \cdot 5 = 300$ possible options for Rumi's room so far. However, rooms 4028 and 4029 are occupied. Note that 4028 is even and is not even on the list, but 4029 fits all of the clues. So, we need to subtract 1 from our total number of rooms. This gives us $300 - 1 = \boxed{299}$.

19. Let $BC = x$, then $AB = x + k$ and $CD = x - k$ for some k, since the lengths form an arithmetic sequence. We are given that $AC = AB + BC = 2x + k = 11$ and $BD = BC + CD = 2x - k = 9$. Solving these equations gives $x = 5$ and $k = 1$. Thus, $BE = BC + CD + DE = x + (x - k) + (x - 2k) = 3x - 3k = 15 - 3 = \boxed{12}$.

20. The only digits that can work in the ones place are 4, 6, 8, and 9, and 0. We can find the 3-divisible numbers for each case:

$$4 : 124$$

$$6 : 126, 136$$

$$8 : 128, 148, 248$$

$$9 : 139$$

For the 0 case, since any integer except 0 divides 0, we can count the 2-divisible numbers not ending in 0. This is simply $1 + 1 + 2 + 1 + 3 + 1 + 3 + 2 = 14$. Thus, there are $\boxed{21}$ 3-divisible numbers.

21. The probability he misses both his first and second serves is $\frac{2}{5} \times \frac{1}{10} = \frac{1}{25}$. Since we want to calculate the probability he does not miss both serves, we seek $1 - \frac{1}{25} = \frac{24}{25}$, giving us a final answer of $24 + 25 = \boxed{49}$.

22. Notice that a positive integer has exactly three positive divisors only when it is the square of a prime; if n is our *nearly prime*, and $n = p^2$ where p is a prime number, then the only divisors of n are $1, p$, and n. The primes whose squares lie under 50 are $[2, 3, 5, 7]$. We add their squares, which are $[4, 9, 25, 49]$, to find our sum of $\boxed{87.}$

23. Recognize that the original set up forms a right triangle with vertical leg 6, horizontal leg 8, and hypotenuse 10. We want the vertical leg to measure 5 feet. Using the Pythagorean theorem yields us that the horizontal leg must be $5\sqrt{3}$. The absolute distance is $5\sqrt{3} - 8$, so our answer is $8 + 5 + 3 = \boxed{16}$.

24. Notice that for Joey to be able to make it to $(3, 4)$, he must consecutively go in one direction (up or right) until he is either at $y = 4$ or $x = 3$. In other words, if his movement was modeled in a sequence using "R" for right and "U" for up, he must either have "UUUU" or "RRR" in his sequence. One such valid sequence would be "RUUUURR." There are 3 places to insert "RRR" into U's, and there are 2 places to insert "UUUU" into R's. Thus, there are $\boxed{5}$ total number of paths.

25. The maximum area is achieved when the 6 fence posts are in a hexagonal position, given by the isoperimetric property for polygons with a given perimeter and number of sides. The hexagon can be split into 6 equilateral triangles each with area $\frac{\sqrt{3}}{4}$ yielding a final area of $\frac{3\sqrt{3}}{2}$. Thus, $a + b + c = \boxed{8}$.

26. By symmetry, the longest diagonal of the cube should be equal to the diameter of the sphere. Thus, for side length s of the cube, knowing that $s\sqrt{3}$ represents the longest diagonal of the cube, we have $s\sqrt{3} = 6$. This allows us to find $s = \sqrt{12}$, through which we find $V^2 = s^6 = \boxed{1728}$.

27. The number of positive integers less than 1000 divisible by 2 is 499 (these are the even numbers). The number divisible by 5 is 199 (these include $5, 10, 15, \ldots, 995$). The numbers divisible by both 2 and 5 are the multiples of 10, and there are 99 such numbers less than 1000.

    Therefore, the number of positive integers less than 1000 divisible by exactly one of 2 or 5 is

$$(499 - 99) + (199 - 99) = 400 + 100 = \boxed{500}.$$

28. If Daniel is telling the truth, then Gabriele is telling the truth, which means Daniel is lying. That means Gabriele is telling the truth and Daniel is lying, so Chris must also be lying. Then Arnav must be telling the truth. If Fiona is telling the truth, then Bryan must be lying, but then exactly 3 people would be lying, so Bryan would be telling the truth. Therefore Fiona must be lying, and the answer is $\boxed{ABG}$.

29. Say that initially all of the balls are in the first box, so the difference is $1+2+4+\cdots+1024-0=2047$. If I move some balls with total value X from the first box to the second, the difference decreases by $2X$, because instead of adding X, you are now subtracting it. Since every number has a binary representation and the balls are powers of two, X can be any number from 0 to $1+2+4+\cdots+1024=2047$, and each X gives a unique value of $A-B$. Therefore, there are $\boxed{2048}$ possible values.

30. By the remainder theorem, we know that given quadratic $f(x)$, the remainder of $f(x)$ when divided by $(x-2)$ and $(x+2)$ will be $f(2)$ and $f(-2)$, respectively. Thus, for the remainders to be equal, $f(2)=f(-2)$. Now, given the quadratic term will evaluate equally $(7(2)^2=7(-2)^2)$, and the constant term stays constant between $f(2)$ and $f(-2)$, we know that the coefficient of x in $f(x)$ must be equal to 0 (i.e. $f(x)=7x^2+c$ for some constant c). For $f(2)=7(2)^2+c=8$ to be true, $c=-20$, giving us a final solution of $f(1)=7-20=\boxed{-13}$.

31. The optimal method to explore all the tunnels is for each person to explore a different cave at a time, so every minute we can eliminate 3 options. We know that we must take at most 3 minutes to *know* where the exit is, since after 9 tunnels are explored and the exit has not been found, we know the last cave must be the exit.

    Now there is a $\frac{3}{10}$ chance the first 3 tunnels the group explores on minute 1 has the exit, so we have $\frac{3}{10}\cdot 1=\frac{3}{10}$ minutes. There is a $\frac{3}{10}$ chance the second 3 tunnels the group explore on minute 2 has the exit, so we have $\frac{3}{10}\cdot 2=\frac{6}{10}$ minutes. By complementary counting, the probability we go past minute 2 is $\frac{4}{10}$, but by the logic said before we are guaranteed to find the exit on minute 3 so we have $\frac{4}{10}\cdot 3=\frac{12}{10}$ minutes.

    Summing we get $\frac{3}{10}+\frac{6}{10}+\frac{12}{10}=\frac{21}{10}$ minutes, so $\frac{21}{10}\cdot 60=\boxed{126}$ seconds.

32. It is observable that the 7th overlap will occur around 7 o'clock. The speed of the minute hand is $\frac{360}{60}=6$ degrees per minute, while the speed of the hour hand is $\frac{30}{60}=0.5$ degrees per minute. Let's say our desired time where the 7th overlap occurs is x minutes past 7. We can thus write the equation:

$$6x=210+0.5x$$

    where 210 corresponds to the hour hand's position at 7 o'clock (since $7\times 30=210$ degrees). Solving for x, we get that

$$x\approx\boxed{38}$$

    Therefore, the 7th overlap occurs approximately 38 minutes past 7.

33. The infinite series can be factored into two geometric series as

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\cdots=\left(1+\frac{1}{2}+\frac{1}{2^2}+\dots\right)\left(1+\frac{1}{3}+\frac{1}{3^2}+\dots\right).$$

    Using the formula for infinite geometric series, we find $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\cdots=\frac{1}{1-\frac{1}{2}}\cdot\frac{1}{1-\frac{1}{3}}=2\cdot\frac{3}{2}=\boxed{3}$.

34. Note that the prime factorization for 10! is $2^8\cdot 3^4\cdot 5^2\cdot 7^1$. For a divisor of 10! to be a perfect square, each square must be raised to an even power. If an arbitrary divisor of 10! is $2^{e_2}3^{e_3}5^{e_5}7^{e_7}$, e_2 can be 0, 2, 4, 6, or 8; e_3 can be 0, 2, or 4; e_5 can be 0 or 2; e_7 must be 0. So we have $5\cdot 3\cdot 2\cdot 1=30$ perfect square divisors. The total number of divisors is $(8+1)(4+1)(2+1)(1+1)=270$, so the probability of a divisor being a perfect square is $\frac{30}{270}=\frac{1}{9}$, so $m+n=\boxed{10}$.

35. Let X_{ij} be 1 if person i and person j form a compatible pair and 0 otherwise. Let X be the sum of X_{ij} taken across all unique pairs of people. Each person chooses a recipient uniformly from the other 19 people, so

$$\Pr(X_{ij}=1)=\frac{1}{19}\cdot\frac{1}{19}=\frac{1}{19^2}.$$

With $\binom{20}{2} = 190$ unordered pairs, linearity of expectation gives

$$\mathbb{E}[X] = 190 \cdot \frac{1}{19^2} = \frac{190}{361} = \frac{10}{19}.$$

Thus, the answer is $10 + 19 = \boxed{29}$.

36. For a sum S, Colin is able to deduce at least one of the dice rolls if all triples $1 \leq a, b, c \leq 6$ satisfying $a + b + c = S$ share a common element. We claim that this only occurs when $S \in \{3, 4, 5, 16, 17, 18\}$. Indeed, if $S = 3, 4, 5$, notice that $\min(a, b, c) \leq \frac{S}{3} < 2$ implying that the minimum element of the triple is always 1, so it must be common amongst all such triples. A similar argument holds for $S = 16, 17, 18$, as we require $\max(a, b, c) \geq \frac{S}{3} > 5$ so $\max(a, b, c) = 6$ and 6 would be common in all triples.

    Now we show that $6 \leq S \leq 15$ fail. It suffices to find two triples that have no common elements that sum to S.

    For $6 \leq S \leq 10$, we can choose the triples $(a, b, c) = (1, 1, S - 2), (2, 2, S - 4)$ and clearly $1 \neq S - 4$ and $S - 2 \neq 2$ so no common elements are shared between these sets.

    For $11 \leq S \leq 15$, we can choose the triples $(a, b, c) = (5, 5, S - 10), (6, 6, S - 12)$ and clearly $5 \neq S - 12$ and $S - 10 \neq 6$ so no common elements are shared between these sets.

    The rest is simple counting. For $S = 3$ the only working triple is $(1, 1, 1)$ and permutations, for $S = 4$ we have $(1, 1, 2)$ and permutations, and finally for $S = 5$ we have $(1, 1, 3), (1, 2, 2)$, and permutations. Altogether for $S = 3, 4, 5$, we have $1 + \binom{3}{1} + \binom{3}{1} + \binom{3}{1} = 10$ possibilities. Now we have a bijection $(a, b, c) \iff (7 - a, 7 - b, 7 - c)$ from a sum S to $21 - S$, so the number of cases from $S = 16, 17, 18$, is identical. So we have $10 \cdot 2 = 20$ working cases.

    Since there are $6^3 = 216$ total triples, our answer is $\frac{20}{216} = \frac{5}{54} \implies 5 + 54 = \boxed{59}$.

37. Since we know the side length of the pyramid's base, and thus the base's area, we simply need to find the pyramid's height. Consider the cross section of the sphere inside the pyramid. The radius is 1, so $CD = DE = 1$. BC is half of the base, so $BC = 2$. Let $AD = x$. Since the circle is tangent to each of the sides of the pyramid, $AE \perp ED$, and $AC \perp CB$. We can now use the Pythagorean Theorem on $\triangle AED : AE^2 + 1^2 = x^2$, so $AE = \sqrt{x^2 - 1}$. Now, since they share $\angle A$, right triangles AED and ACB are similar. So, $\frac{AE}{ED} = \frac{AC}{CB}$, or $\frac{\sqrt{x^2 - 1}}{1} = \frac{x + 1}{2}$. The positive solution for x is $\frac{5}{3}$, so $AC = x + 1 = \frac{8}{3}$. Finally, the volume of the pyramid is $\left(\frac{1}{3}\right)\left(\frac{8}{3}\right)(4^2) = \frac{128}{9}$, so $m + n = \boxed{137}$.

38. Define the number of terminal zeroes to the be number of consecutive zeroes at the end of the decimal representation of a given number. There are 45 textbooks in total, and to get a unique arrangement, we choose 21 of the spaces on the shelf to be geometry textbooks. The rest will have to be algebra textbooks. This gives us $N = \binom{45}{21} = \frac{45!}{21! \cdot 24!}$.

    The number of terminal zeroes is equivalent to the number of times 10 divides N. To find this, we want to find the number of powers of 2 and 5 in the prime factorization of N. Let $\nu_p(n)$ denote the number of powers of a prime p in the prime factorization of an integer n. Note that by exponent rules, $\nu_p(ab) = \nu_p(a) + \nu_p(b)$ and $\nu_p(a/b) = \nu_p(a) - \nu_p(b)$. We use Legendre's formula ($\nu_p(n!) = \sum_{k \geq 1} \lfloor n/p^k \rfloor$) to get

$$\nu_2(N) = \nu_2(45!) - \nu_2(21!) - \nu_2(24!) = 41 - 22 - 18 = 1$$
$$\nu_5(N) = \nu_5(45!) - \nu_5(21!) - \nu_5(24!) = 10 - 4 - 4 = 2.$$

    Thus, the prime factorization of N looks like $2^1 \cdot 5^2 \cdot$ (other stuff). The answer is $\min\{\nu_2(N), \nu_5(N)\} = \boxed{1}$.

39. Let $Q(x) = P(x) - 2(5 - x)$. Then, for $x = 0, 1, \cdots, 5$, $Q(x) = 0$. So $Q(x)$ has roots at 0 through 5. Since $Q(x)$ is also a degree 6 polynomial with leading coefficient 1, we have

$$Q(x) = x(x - 1)(x - 2)(x - 3)(x - 4)(x - 5).$$

    To find $P(6)$,

$$Q(6) = P(6) - 2(5 - 6) = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6! = 720.$$

    Solving for $P(6)$, we get $P(6) = \boxed{718}$.

40. Label the columns by 1 to 5 from left to right, and label the rows by 1 to 5 from bottom to top. Let cell (i, j) be the cell in column i and row j.

    First, we can color column 1 arbitrarily in 2^5 ways. In column 2, we can color cell $(2, 1)$ either black or white. In either case, the rest of column 2 is forced by repeatedly applying the 2×2 condition. We can keep performing this algorithm of deciding black or white for each of the proceeding columns.

    Since we have 2^5 ways to color column 1, and 2^4 ways to color the other 4 columns by deciding the color of the bottommost cell, we have $2^{5+4} = 2^9 = 512$.

    Thus, there are $\boxed{512}$ ways.